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SIMULATION OF KALMAN FILTER USED FOR
LONG BASELINE UNDERWATER TRACKING

by

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A Kalman Filter that processes individual range measurements one at a time is used to simulate performance of a long baseline system. The state vector includes hydrophone locations.

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I. INTRODUCTION

The conventional method for tracking a target using range measurements from a field of sensors involves the selection of three such sensors to fix position by doing the algebraic equivalent of finding the intersection of three spheres. No a priori knowledge of the position or velocity of the target is required except to select one of the two points in the intersection, and even this can be avoided by utilizing a fourth sensor. Each position estimate is made independently of all other position estimates.

In Kalman filtering, on the other hand, it is not necessary to employ exactly three sensors; position estimates simply become more accurate as the number of sensors measuring a range is increased. Even if the number of sensors is 0, a Kalman filter will measure (inaccurately) the target's position. The reason for this is that a Kalman filter operates by continually modifying a previous estimate, with the modification being 0 in case there is no conflict between the estimate and the observed measurements. If the estimate is bad, the modified estimate will also probably be bad; in this sense, successive estimates are not independent. In any case, one consequence of using a Kalman filter for tracking is that an initial estimate of the object's position (and velocity, in our simulation) must be provided to the filter.

One purpose of our simulation is to test how well a Kalman filter does at tracking an object through a field of seven sensors hypothetically arranged as in Figure 1, with each sensor measuring range if the range to the object is less than 1000 yards (the circles in Figure 1 have a radius of 1250 yards). Another purpose is to see

how well the filter does when the positions of the sensors, as well as the position of the target, are known exactly. This latter problem is felt to be an important one, particularly in applications where the sensor field is temporary, or subject to continual component replacement. The filter is suppose to simultaneously estimate sensor and target positions, with the estimates of sensor positions presumeably converging to the true positions with time (the true position is unknown but stationary).

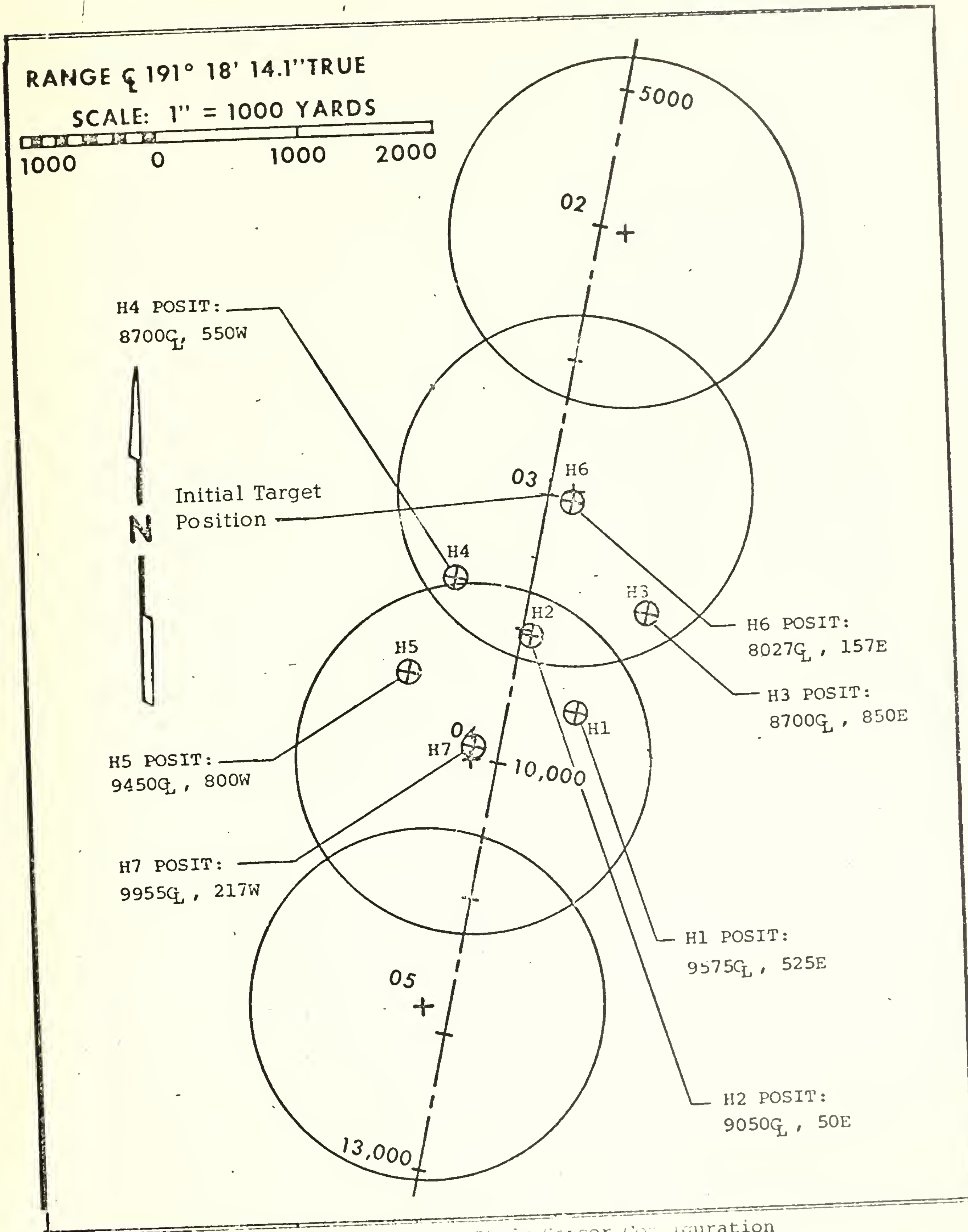

Figure 1 NPS Tracking Study Sensor Configuration

2. RESULTS

For the first problem, where sensor positions are known exactly, the error in estimating the target's position, after the effects of the assumed bad initial estimate have damped out, seems to be roughly three times the error in measuring range. There can be nothing universal about the number "three", since it obviously depends on sensor configuration and density; nonetheless, it is somewhat encouraging that error magnification is not of the order of 100 or 1000, as it can be in short baseline systems.

For the second problem, not much can be said in general except that the filter does a better job of estimating the target's position at the end of one "pass" through the field that it does of estimating the positions of the sensors. Intuitively, the reason for this is that the target is involved in every range measurement, whereas the typical sensor is involved in only a few. Further details are in the text.

3. DEFINITIONS AND DESCRIPTIONS

A Kalman filter can be viewed either as the solution of an optimization problem, or in the context of a conditional Bayesian estimation problem. We adopt the latter viewpoint, in which case $\hat{x}$ and P in the following can be interpreted as the (vector) mean and covariance matrix of the state, conditional on all past observations. Initial values of $\hat{x}$ and P must be supplied, and one of our purposes is to explore sensitivity of results to these initial estimates. In our problem, there are six coordinates for the target (position and velocity), and three for each of the seven sensors, so x (the true state vector, of which $\hat{x}$ is an estimate) and $\hat{x}$ are both 27-vectors, and P is a 27×27 matrix.

In all cases, the initial P-matrix consists of 0's except on the diagonal, indicating that all 27 initial estimates are independent. We will refer to the diagonal of the P-matrix as Var. The initial x, $\hat{x}$, and Var in the baseline case are shown in Table 1. These initial values are supposed to represent the situation just after a field of sensors has been implanted, and just as a torpedo is being launched at true velocity (30, 0, 0) yards/sec. from true initial position (0, 0, 0). The origin for our purposes is on the centerline of Figure 1 at 8000 yds (very near H6), and all distances are in yards. The X-coordinate of H1 is actually 1575 (add 8000 to place it on Figure 1), but is thought to be at 1603, a 28 yard error. The process of implanting hydrophones is known (we assume) to produce errors of about $\sqrt{400} = 20$ yards in the X and Y directions, but only $\sqrt{4} = 2$ yards in the Z direction. Thus, roughly speaking,

INITIAL VALUES FOR THE BASELINE CASE

	Velocity			Position			H1			H2			H3			H4			H5			H6			H7		
x	30	0	0	0	0	0	1575	-525	195	1050	-50	200	700	-850	190	700	550	190	1450	800	185	27	-157	190	1955	217	200
$\hat{x}$	25	5	0	100	5	0	1603	-512	195	1054	-85	201	737	-825	187	685	507	189	1477	795	185	25	-172	191	1941	220	201
Var	100	100	20	9999	1000	100	400	400	4	400	400	4	400	400	4	400	400	4	400	400	4	400	400	4	400	400	4

TABLE I

the initial estimate of x_1 is 1550 ± 20, with a similar interpretation in the other 26 columns. Note that velocity in the X-direction is estimated to be 25 ± $\sqrt{100}$, and is actually 30, and that the initial estimate of the X-coordinate of the target's position is off by 100 yards. We shall generally refer to columns with small values of Var as "precise", and the columns where $|x-\hat{x}|$ is small as "accurate".

Successive states x are all identical except that 30 is added to X_4 every second; the target actually proceeds in a straight line on the surface on the centerline. In the simulation, a subroutine outside of the Kalman filter computes the seven true ranges, adds noise to them, and presents those that are $\leq$ 1000 yards to the filter for processing to obtain the next estimate $\hat{x}$ and the next Var. The noise (error in measuring true range R) is assumed to have mean 0 and standard deviation .5 + .0005R yards in the baseline case. The .5 is supposed to represent jitter and any other sources of error that are independent of range, and the .0005R is supposed to model the effects of random variations in the sound speed profile, which cause large range measurements to be more in error more than short ones. Note that the two sources of error are equal at R = 1000 yards. This formula (but not the individual errors) is in the filter, and the effect is to cause the filter to mistrust large range measurements. The output of the simulation is just x, $\hat{x}$, and Var at successive instants of time (the filter actually keeps track of the whole covariance matrix P, but only the diagonal is output), along with the "Kalman gains" for each range measurement, etc.

There is another important input to the filter. Any position estimating scheme that "updates" partially by asking "Are these new measurements reasonably consistent with what has previously been estimated about the state?" has got to deal quantitatively with the word "reasonable". In this simulation, the filter assumes that the target actually moves by making random increments to its velocity every second, and position changes are "reasonable" if they don't imply velocity increments larger than the standard deviations involved. The three standard deviations are all $\sqrt{50}$ yards/sec., which are supposed to be characteristic of a high speed torpedo making turns at 10°/sec. Since the target in this simulation actually moves in a straight line, the accuracy results to be presented later could all be improved by using a number smaller than $\sqrt{50}$. However, the filter would then perform poorly on tracks that were not straight lines. Thus, it is the kind of accelerations to be expected that have determined $\sqrt{50}$, rather than the (null) accelerations actually characteristic of the sample track.

4. SOME DETAILS OF THE FILTER

Imagine that the target produces a "beep" every second at a known time. The several hydrophones will observe pulses in groups, with the groups arriving once a second. The position estimates discussed later are estimates after each group is processed. Nonetheless, the pulses are actually processed one at a time, and position estimates are available after each new pulse. Processing the pulses one at a time is particularly easy computationally, since no matrix arithmetic is required (exclusive of input and output statements, the entire simulation requires about 50 FORTRAN lines).

In Kalman filtering, the measurements are supposed to be _linear_ functions of the state variables. This is not so in measuring ranges, since a sphere is not a linear surface. In the simulation, this problem is taken care of by replacing the sphere with a tangent plane, with the point of tangency being in the same direction as the predicted position. This works all right if errors are not too large, but we will show one example where the filter loses track of the target completely, after which reacquisition is unlikely because the linearization is grossly wrong.

5. EXPERIMENTS

a. Experiment 1: Baseline Except Sensor Positions Known

Figure 2 shows RMS position error (absolute position error) as a function of time and Figure 3 shows $\sqrt{Var_6}$ as a function of time. Both figures have a minimum in the middle when the target is within range of several sensors; in fact, note that three sensors is still a magic number, in the sense that errors tend to be relatively large when less than three sensors respond. Var_6 (vertical variance) is larger than Var_4 and Var_5, the reason no doubt being that range measurements are much more sensitive to the large X,Y distances than to the small Z distance. Note that the RMS error is of the same order of magnitude as $\sqrt{Var_6}$.

There are two reasons for error here: The noisy range measurements and the incorrect initial target position. The effects of the latter are small after the first few seconds. In a different run, the effects of measuring ranges 10 times as accurately were explored. The results were not surprising: Errors were reduced by about a factor of 10.

b. Experiment 2: Sensor Positions Unknown

The RMS position error for the baseline case is shown in Figure 4. It decreases to a minimum before increasing again as the target leaves the sensor field in Runs 1 and 2 (different range errors). X-Y motion plots are shown for sensors H5 and H6 (the Z motions are very small because of the small input variances). Hydrophone H5 remains stationary (we mean that the estimate of

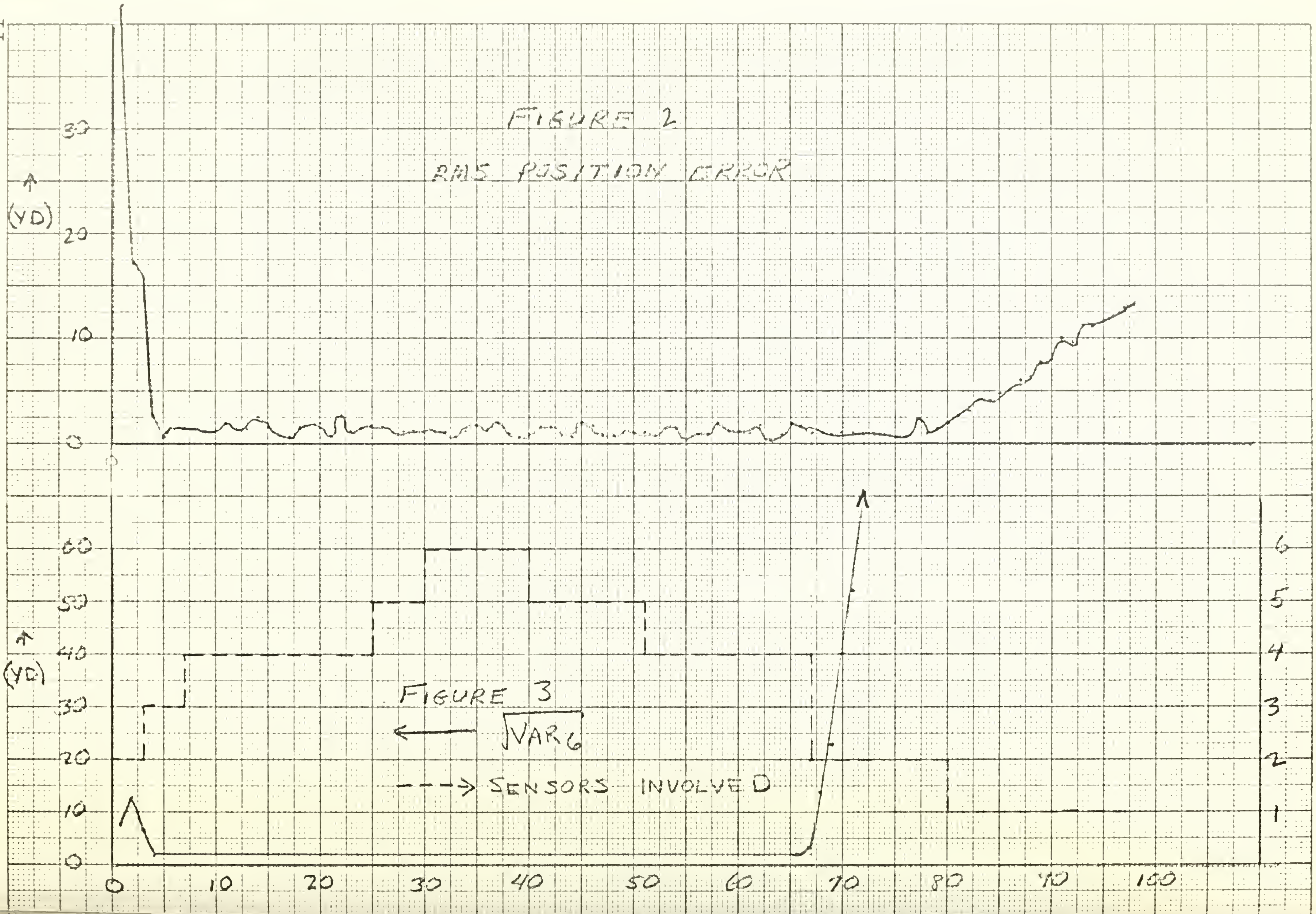
FIGURE 2
RMS POSITION ERROR
(YD)
FIGURE 3
√VAR6
SENSORS INVOLVED
(YD)

its position does not change) for the first 900 yards of target motion ($X_4 \leq 900$), after which the estimated position gradually approaches the true position (the origin in Figure 5). The behavior was similar in both runs. Hydrophone H6 does not behave so nicely; note that the position estimate is worse at the end of the run than at the beginning. The poor Y-estimate has a simple explanation; H6 is so close to the centerline that no range measurement is sensitive to it. Presumably a second target run on a different line would improve the estimate of H6.

The RMS errors differ strongly between the first and second run, as shown in Figure 4. The Var vector, however, changes very little from run to run. The fact that it changes at all can be blamed on the linearization of the measurement equations; the Var computations would not depend on measurements at all in a truly linear system. At time 70, for example, the two Var vectors are the first two rows of Table II.

Note that

1) Hydrophones well off the track of the target have the most precise Y-coordinate estimates.

2) Most of the target position error is in the Z-coordinate the estimate of which has a standard deviation of about $\sqrt{2200} = 47$ yards.

In Run 3, the baseline case was modified by initially locating H4 with perfect accuracy and precision. The effect can be seen in the third line of Table II; the variances are in all cases reduced, with

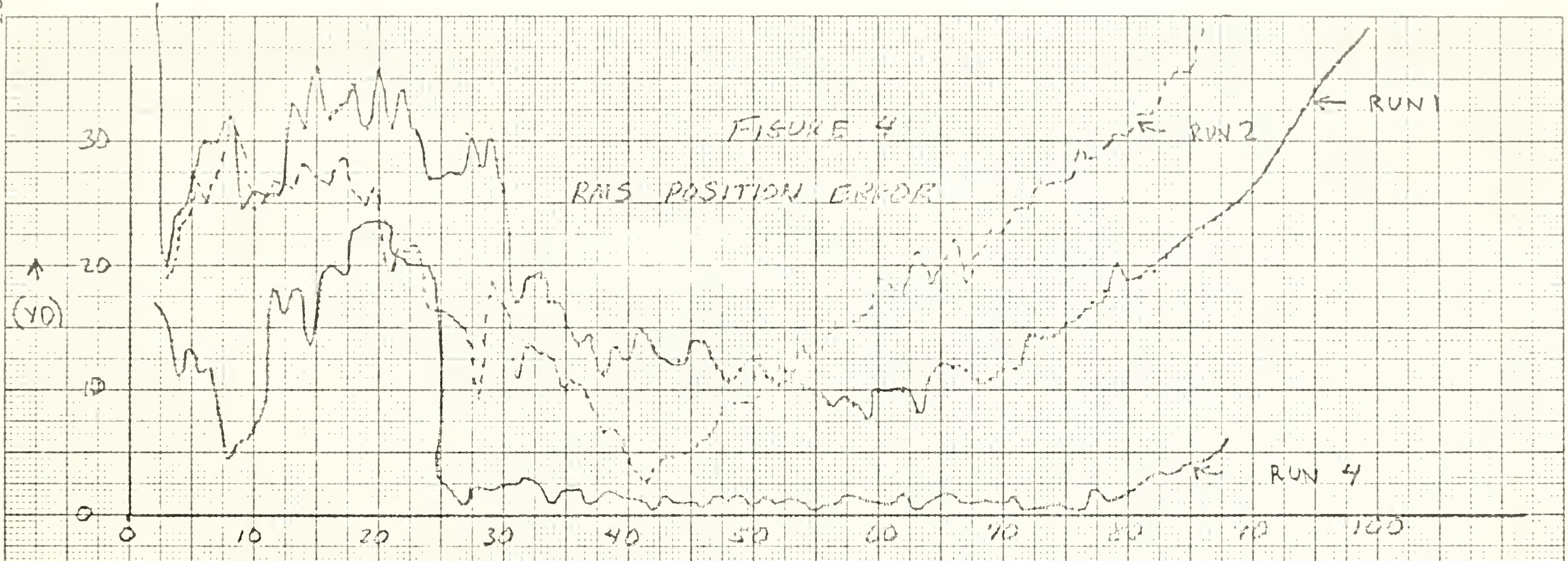

FIGURE 4
RMS POSITION ERROR

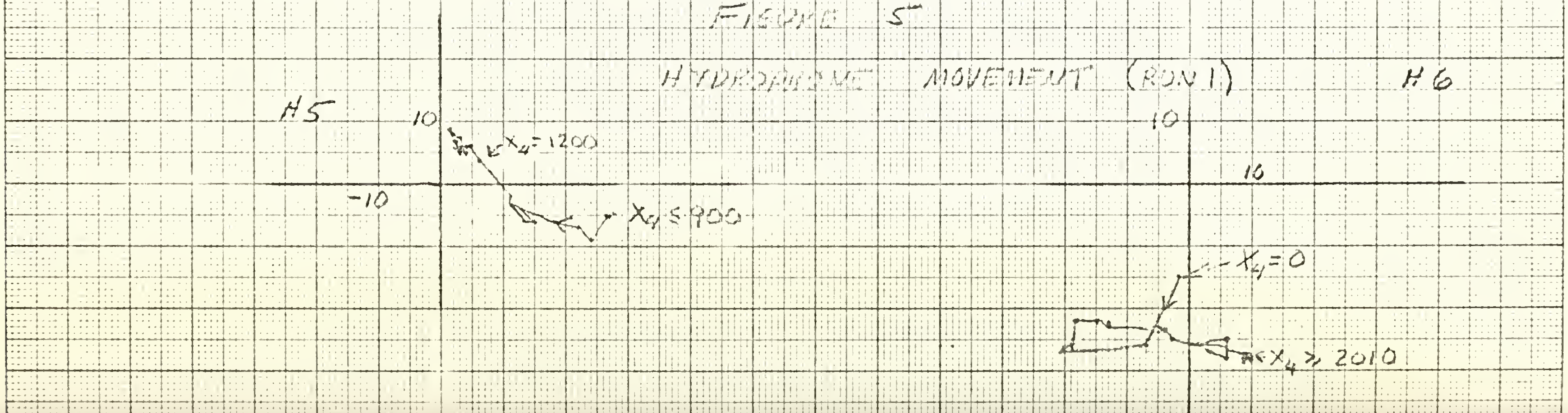

FIGURE 5
HYDROPHONE MOVEMENT (RUN 1)

	Velocity			Position			H1			H2			H3			H4			H5			H6			H7		
Run 1	243	212	251	1510	905	2236	94	151	4	61	276	4	131	73	4	104	108	4	117	92	4	71	209	4	66	323	4
Run 2	245	213	248	1586	931	2096	92	154	4	60	291	4	127	73	4	102	108	4	114	94	4	70	209	4	68	331	4
Run 3	208	166	234	1043	373	1892	88	68	4	31	268	4	120	16	4	0	0	0	10	32	4	35	183	4	38	314	4
Run 4	202	165	240	910	247	1522	0	0	0	0	40	3	1	0	4	0	0	0	1	0	4	0	0	0	0	0	0

Table II

Variance of the Estimates in Several Runs At Time 70

the effect being strongest on those hydrophone nearest H4. In Run 4, H1, H4, H6, and H7 were all located with perfect accuracy and precision. The idea is to model the insertion of three new sensors (H2, H3, H5) into an old, well located field. The steep plunge in the RMS error curve occurs at the first instant when the target is within range of three old sensors. At the end of Run 4, the terminal (X,Y,Z) errors for H2, H3, and H5, are respectively, (0.5, 0.0,-0.2), (-1.0, 1.1, -2.9), and (0.7, -0.1, 0.0).

In Run 5, the baseline case was modified by changing the initial Variances to 100,000 except for the three velocity estimates. Since $\sqrt{100{,}000} \approx 320$ yards, and since 320 yards is not small compared to the range of the sensors, there is the possibility that serious errors will be committed in the spherical linearization step. The RMS error is shown in Figure 6. Estimates up to time 30 are not too bad on account of the fact that initial position estimates are actually reasonably accurate. However, the low precision of the initial estimates causes the filter to change the sensor positions great distances merely on the basis of noise. After time 40, the filter can no longer be said to be "in control"; RMS error becomes larger and larger and sensor estimates become so inaccurate that further range measurements simply lead to more confusion. Evidently, 320 yards is too large a positioning error to be acceptable when the hydrophones have a range of 1,000 yards.

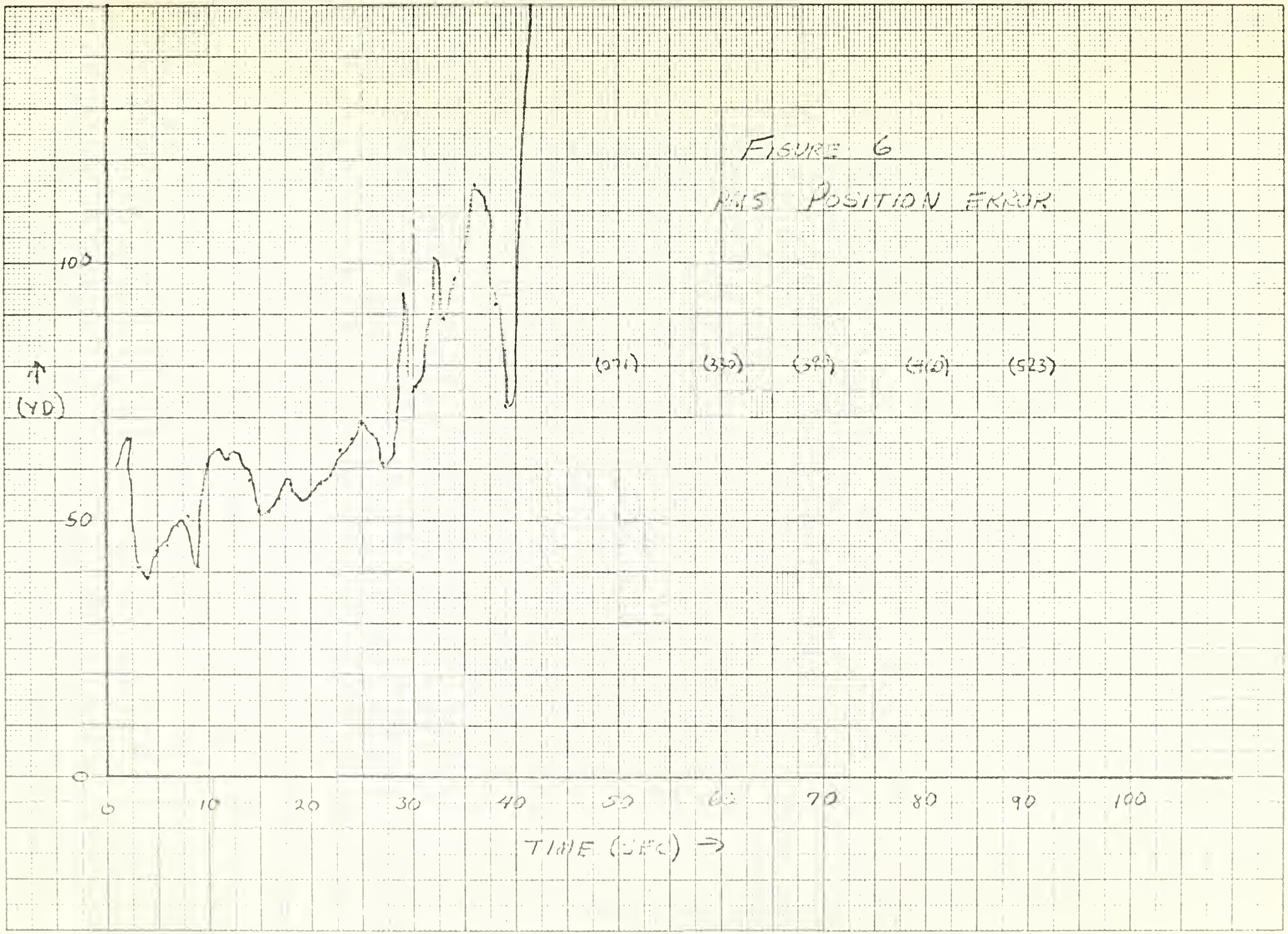
FIGURE 6
RMS POSITION ERROR
(YD)
100
50
0
(523)
0
10
20
30
40
70
80
90
100
TIME

6. OTHER ISSUES

The experiments done so far demonstrate that a Kalman Filter can be used for long baseline tracking, provided that sensors can be implanted reasonably accurately in the first place, and provided that measurement errors can be modeled as white noise. In regard to the white noise assumption, two questions arise:

a. What happens if range measurements are actually biased (consistent errors)?

b. What happens if there are occasional very large "measurement errors" caused by something other than the target pulsing a hydrophone? This question is tied up with selection of hydrophone range, since range can always be increased if one is willing to accept more errors of this type. Perhaps a simple 3σ gate around the estimated range could be used to eliminate such outliers.

Some experiments in which the data rate is decreased from one pulse per sec would also be interesting, particularly in a system where the hydrophone range is considerably larger than 1,000 yards. The beneficial effect of telemetering depth as measured by a pressure sensor on board the target could also be quantified, although the filter would have to be modified order to accommodate such measurement.

It is intended that issues such as those mentioned above will be dealt with in future work.

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[1] Stochastic Optimal Linear Estimation and Control, J. Meditch, McGraw Hill (1969).

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